Section 4.3 Absolute Value Equations and Inequalities

The definition of absolute value: $|\bigstar| = \begin{cases} -\bigstar & \text{if } \bigstar < 0 \\ \bigstar & \text{if } \bigstar \geq 0 \end{cases}$

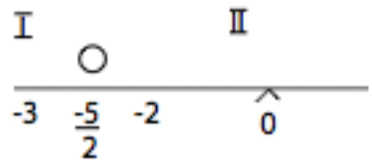
The first thing to notice that is different than the textbook is that we use stars in our definition rather than the x . The star represents "whatever is on the inside". The reason this is significant is that x could be negative but the inside could still be positive. (For example if $x + 4$ is on the inside, then when $x = -3$, the expression $x + 4$ is still positive.) We do not really care if x is negative or not. What concerns us is if the expression inside the absolute bars is negative or not.

Why do we care in the first place?

Absolute value, as we were introduced to it in chapter 1, is the distance from zero --- a non-negative quantity. Absolute values do not behave nicely in algebraic equations. The proper way to handle absolute values (especially when they are) in equations is to first remove them and then solve the resulting purely algebraic equations.

Example 2A page 243 done properly:

$|2x+5| = 13$ We *always* draw a number line to illustrate the intervals.



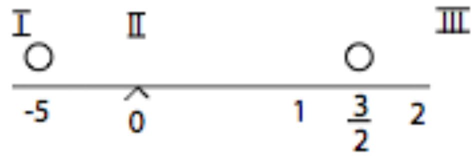
The two intervals are: $x < -\frac{5}{2}$ and $x \geq \frac{5}{2}$

If $x < -\frac{5}{2}$	or	If $x \geq -\frac{5}{2}$
$-(2x+5) = 13$		$(2x+5) = 13$
$-2x - 5 = 13$		$2x + 5 = 13$
$-2x = 18$		$2x = 8$
$x = -9$		$x = 4$
is $-9 < -\frac{5}{2}$		is $4 \geq -\frac{5}{2}$
yes. valid answer.		yes. valid answer.

$x \in \{-9, 4\}$

We use the "interval method" because not only will simple equations, such as this, work nicely but also far more complicated equations will work perfectly using this same method.

We replace example 5 in the text with:



$|2x - 3| - x + 1 = |x + 5|$

The intervals are $x < -5$ $\quad -5 \le x < \frac{3}{2}$ $\quad$ and $x \ge \frac{3}{2}$

If $x < -5$

$-(2x - 3) - x + 1 = -(x + 5)$

$-2x + 3 - x + 1 = -x - 5$

$-3x + 4 = -x - 5$

$9 = 2x$

$\frac{9}{2} = x$

Reject. not in interval

If $-5 \le x < \frac{3}{2}$

$-(2x - 3) - x + 1 = x + 5$

$-2x + 3 - x + 1 = x + 5$

$-3x + 4 = x + 5$

$-1 = 4x$

$-\frac{1}{4} = x$

valid

If $x \ge \frac{3}{2}$

$(2x - 3) - x + 1 = x + 5$

$2x - 3 - x + 1 = x + 5$

$x - 2 = x + 5$

$-2 = 5$

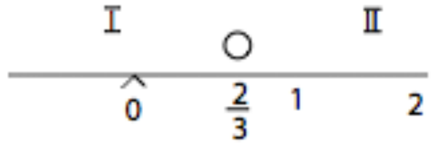
Reject. False.

the only solution to this problem is $x = -\frac{1}{4}$

Notice that **only** those “zeros” of the insides of the absolute values were marked on our number line.

This original equation could have been written as: $|2x - 3| + (1 - x) = |x + 2|$ and its solution would have been exactly the same as we did here.

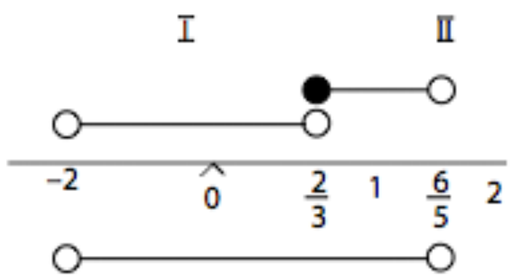
The replacement for example 8: $|3x-2| < 4-2x$

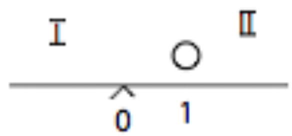


Our two intervals are $x < \frac{2}{3}$ and $x \geq \frac{2}{3}$

If $x < \frac{2}{3}$	If $x \geq \frac{2}{3}$
$-(3x-2) < 4-2x$	$(3x-2) < 4-2x$
$-3x+2 < 4-2x$	$3x-2 < 4-2x$
$-2 < x$	$5x < 6$
	$x < \frac{6}{5}$
is -2 in interval?	is $\frac{6}{5}$ in interval?
Yes, so	Yes, so
$-2 < x < \frac{2}{3}$	$\frac{2}{3} \leq x < \frac{6}{5}$

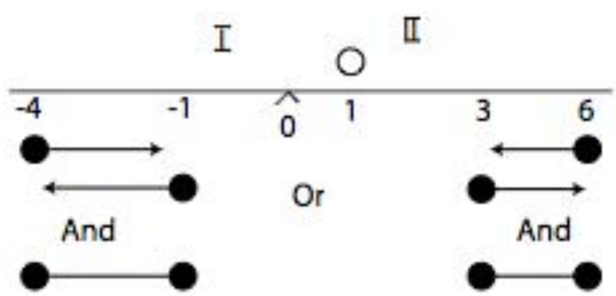
The conclusion is $-2 < x < \frac{6}{5}$





Example: $2 \le |x-1| \le 5$

$2 \le \|x-1\|$	and	$\|x-1\| \le 5$
	If $x < 1$	
$2 \le -(x-1)$		$-(x-1) \le 5$
$2 \le -x+1$		$-x+1 \le 5$
$1 \le -x$		$-x \le 4$
$-1 \ge x$		$x \ge -4$
	OR	
	If $x \ge 1$	
$2 \le x-1$		$x-1 \le 5$
$3 \le x$	and	$x \le 6$



The solution is written as one of these:

$$x \in \{[-4,-1] or [3,6]\}$$

$$[-4,-1] \cup [3,6]$$

The square brackets are used because of the equality.

These problems are to be done ***after*** the class lecture for section 4.3.

For each problem, make a number line showing zero. Indicate zeros of any absolute value expressions. Label intervals using roman numerals and write the specific interval being used in proper format.

For each interval, properly remove the absolute value and solve the resulting equation. Determine the suitability of the solution and finally, draw another number line showing the final solution and write the algebraic expression for that/those interval(s).

A) $|x+1|+|x-2| \leq 7$

B) $|2x+1|+|2-x| \leq 7$

C) $|x+1|-|2x-1| \leq -4$

D) $|2x|-|x+2| \geq 3$

E) $|x+2| < |x-3|$

F) $|x+2|+2x \geq |x-3|$

Algebraic solutions on next page.

A) $-3 \le x \le 4$

B) $-2 \le x \le \frac{8}{3}$

C) $x \le -2$ or $x \ge 6$

D) $x \le -\frac{5}{3}$ or $x \ge 5$

E) $x < \frac{1}{2}$

F) $x \ge \frac{1}{4}$